

Common Ownership and the Secular Stagnation Hypothesis:

Note on Calibration

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Introduction

In this note, we provide more details on the calibration of the model, which for reasons of space, could not be included in the article. We provide the calibration for illustrative purposes, as there is no consensus in the literature on the level or the evolution over time of some of the key ingredients necessary, especially on the average levels of concentration in product and labor markets at the macroeconomic level. We argue that calibrating the model is still useful to illustrate how the model can be brought to the data.

To calibrate the model and obtain the evolution of the labor and capital shares implied by the model, we need the following ingredients:

1. An estimate of the elasticity of labor supply η .
2. An estimate of the elasticity of substitution across sectors θ .
3. The number of sectors N .
4. The average level of concentration in product markets, labor, and capital markets, measured as the modified HHI, $H_{product}$, H_{labor} , and $H_{capital}$.

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5. The average level of the Edgeworth sympathy coefficient within and across industries, λ_{intra} and λ_{inter} .
6. The production function parameter α and the depreciation rate δ .
7. The elasticity of intertemporal substitution $1/\gamma$, and the time preference parameter β .
8. The initial endowment of the savers E , the total-factor-productivity parameter A , and the scale parameter of labor supply χ .

Preference, Technological, and Endowment Parameters

To calibrate η , we assume that the elasticity of labor supply is equal to 0.59, following [Chetty et al. \(2011\)](#). [Manning \(2003\)](#) reports elasticities of labor supply at the firm level between 0.68 and 1.38. [Webber \(2015\)](#) estimates the firm-level elasticity using Census data and obtains an average firm-level elasticity of 1.08. The firm-level elasticities are an upper bound for the sector level elasticities, and therefore using 0.59 seems reasonable.

To calibrate θ , we use estimates from [Hobijn and Nechio \(2015\)](#), who obtain sector level elasticities of substitution under various levels of aggregation, and find elasticities between 1 and 3. Given that the number of sectors in their most disaggregated definition is 73, which is closer to the number of sectors that we will use for the calculation of the average concentration level, we use an elasticity of 3 which is what they find in that specification. [Redding and Weinstein \(2018\)](#) find a median elasticity of substitution at the product level (which should be higher than the sector-level elasticity of substitution) of 4.5.

We set the number of sectors to $N = 453$ based on the number of 4-digit SIC industries in the Compustat database. The large number of sectors implies that the sector-level elasticity of product demand is very close to θ , as the term $1 - 1/N$ becomes close to one.

We calibrate γ to $1/2$, based on the estimate of the intertemporal elasticity of substitution by [Gruber \(2013\)](#). We calibrate the savers' endowment E and productivity A to match the real interest rate of 1.071 in 1985 and the level of capital per worker in that year. We calibrate χ to match the employment-population ratio in 1985. We use $\alpha = 2/3$, $\delta = 0.1$, and $\beta = 0.99$, which are values commonly used in the literature.

Concentration and Common Ownership Parameters

We calibrate the product, labor, and capital market HHIs based on Compustat. The product market HHIs are calculated using 4-digit SIC sector national revenue shares. The labor market HHIs are calculated using 4-digit SIC sector national employment shares. This amounts to assuming that the labor markets are segmented by industry. With segmented markets, the solution to the model is analogous to that derived in the paper, but with market concentration measures based on industry employment shares instead of overall shares. We assume that the capital market is not segmented across industries, and therefore capital market HHIs are calculated using revenue shares across all industries. Not surprisingly, the capital market HHI is almost zero in all years.

We calibrate the common ownership parameters to match our estimates of the Edgeworth sympathy coefficients λ_{intra} and λ_{inter} , which we estimate based on Thomson 13F Institutional Ownership data. We first calculate the sympathy coefficients for every firm pair among the largest 1,500 firms by market capitalization, using the formula from [O'Brien and Salop \(2000\)](#) and assuming proportional control:

$$\lambda_{jk} = \frac{\sum_{i=1}^I \gamma_{ij} \beta_{ik}}{\sum_{i=1}^I \gamma_{ij} \beta_{ij}},$$

where γ_{ij} is the control share of shareholder i in firm j , I the number of investors, and β_{ij} is the financial interest share of shareholder i in firm j . We then estimate λ_{intra} for each year by taking a weighted average of the pairwise sympathy coefficients across all

firm pairs that are in the same 4-digit SIC industry (where the weights are given by the product of the market capitalization of both firms). Similarly, we estimate λ_{inter} for each year by taking a weighted average of the pairwise sympathy coefficients across all firm pairs that are in different industries.

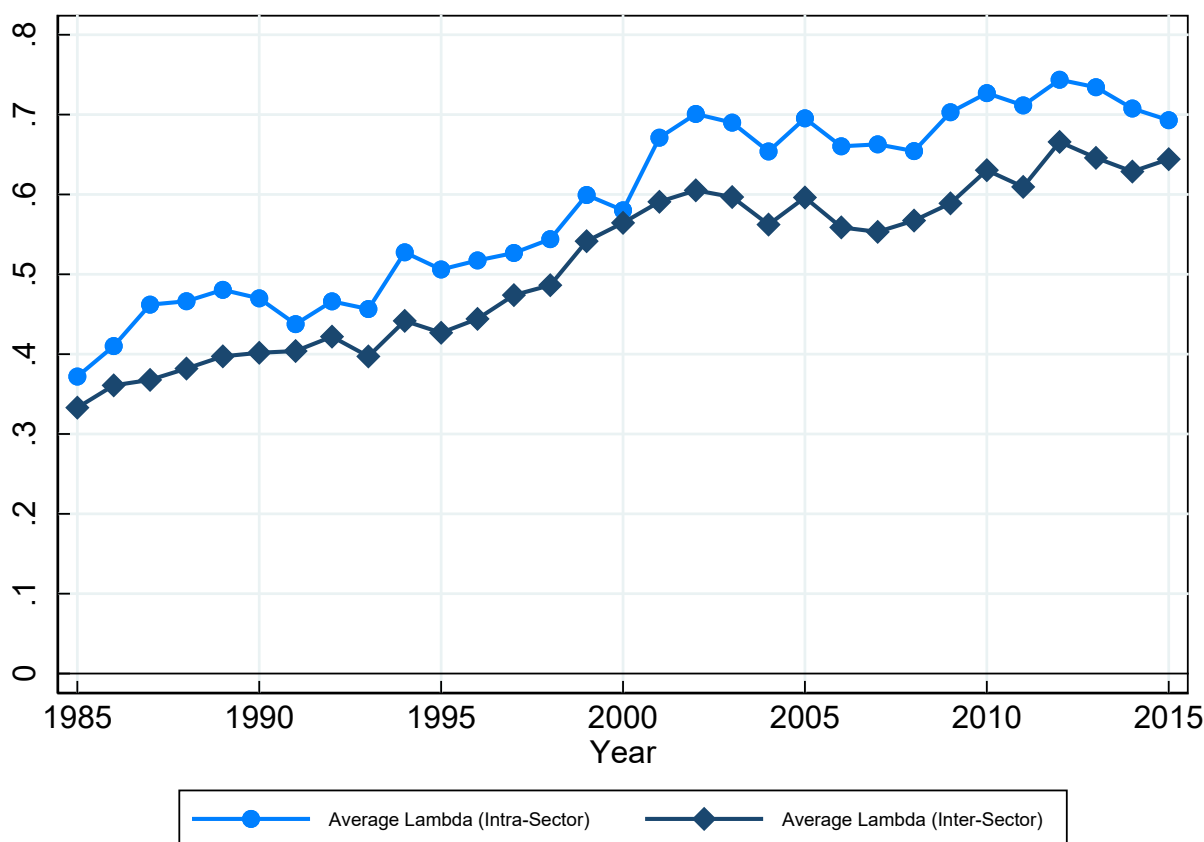


Figure 1. Average Intra-Sector and Inter-Sector Edgeworth Sympathy Coefficients Among Publicly Traded Companies. Source: Authors' calculation using Thomson 13F Institutional ownership data.

Figure 1 shows that the average level of sympathy among publicly traded companies is very high, both intra-industry and inter-industry. The intra-industry average sympathy coefficients went from around 0.4 in 1985 to around 0.7 in 2015. The inter-industry average sympathy coefficient went from around 0.3 in 1985 to around 0.65 in 2015.

These numbers, however, apply only to publicly traded firms. The share of privately held firms in the economy is substantial. We adjust for this fact by using [Asker, Farre-](#)

Mensa and Ljungqvist (2014)'s estimate that privately held companies account for 58.7% of sales in the U.S. economy. Assuming that there is no common ownership between privately held firms, or between privately held and publicly traded firms, we scale downwards the average λ_{intra} and λ_{inter} by multiplying them by $(1 - 0.587)^2$. The results are shown in Figure 2. This adjustment results in much lower sympathy coefficients, both intra-industry and inter-industry. In both cases the coefficients go from an average of around 0.05 in 1985 to around 0.1 in 2015.¹

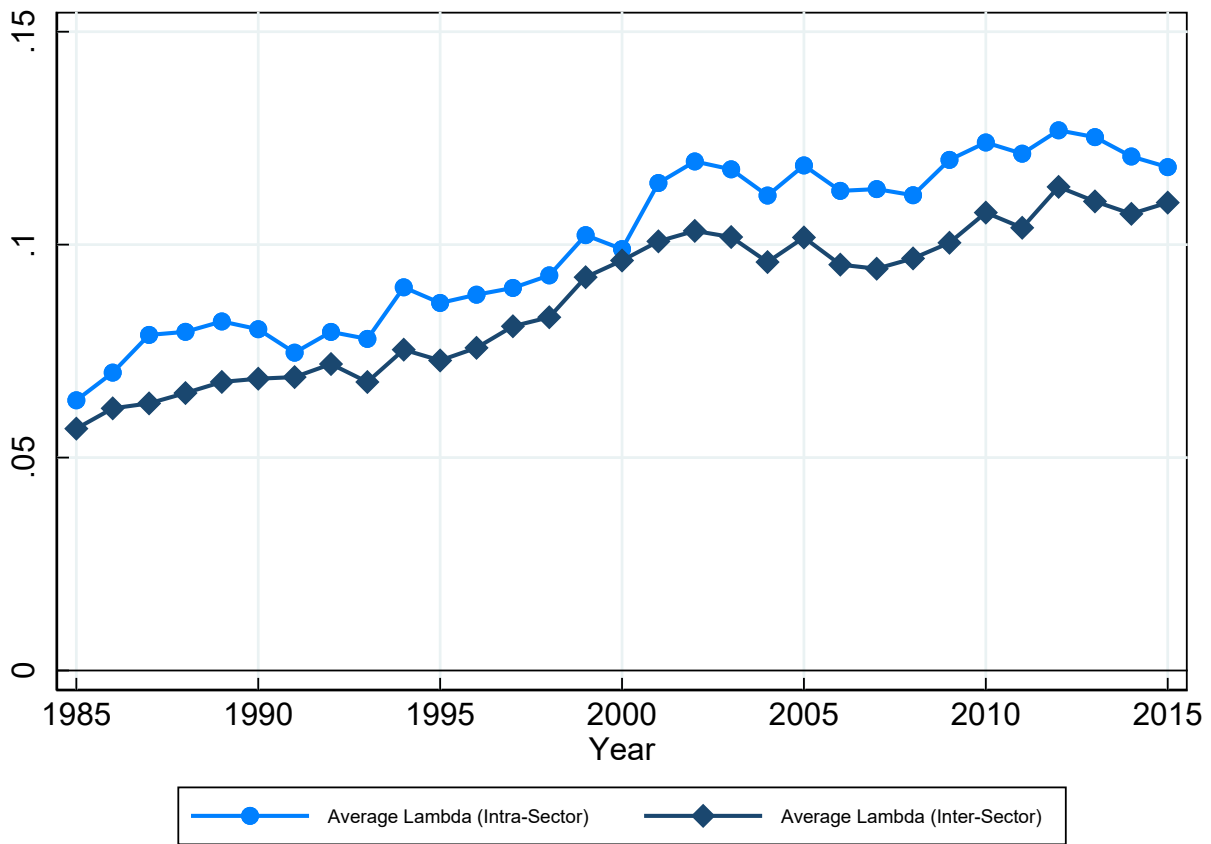


Figure 2. Average Intra-Sector and Inter-Sector Edgeworth Sympathy Coefficients Adjusting for Share of Privately-Held Companies. Source: Authors' calculation using Thomson 13F Institutional ownership data.

¹We can solve for the values for ϕ and $\tilde{\phi}$ implied by these values for λ_{intra} and λ_{inter} . The implied ϕ goes from 0.914 in 1985 to 0.939 in 2015. The implied $\tilde{\phi}$ goes from 0.004 in 1985 to 0.0148 in 2015. The high value of ϕ reflects the fact that the portfolio of the "weighted average shareholder" of the typical firm in the economy has a relatively small fraction invested in that firm, compared to the weight in other firms in the economy. This stylized fact implies a ratio $(1 - \phi)/\phi$ is close to zero, and therefore a ϕ close to one.

We calibrate the average MHHI in product and labor markets as $HHI + \lambda_{intra}(1 - HHI)$ (using the respective HHI), based on our estimate of the average intra-industry Edgeworth sympathy coefficient λ_{intra} . For capital market MHHI delta, we do the same but using $HHI_{capital} + \lambda_{inter}(1 - HHI_{capital})$, which is approximately λ_{inter} .

Figure 3 shows the HHIs and MHHIs for product and labor markets. In both cases, the HHI and the MHHI both increase. The increase in MHHI is much larger than the increase in HHI, indicating that concentration from common ownership has increased more than concentration from higher market shares.

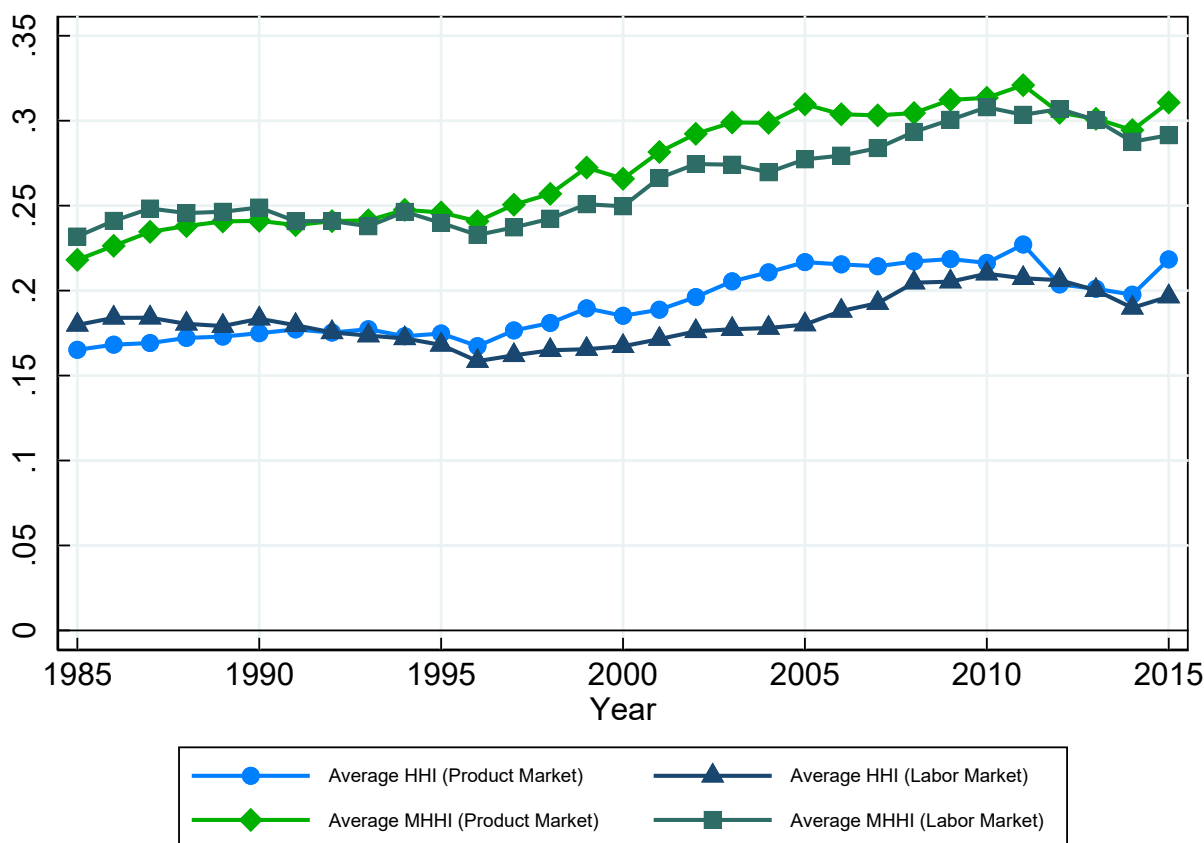


Figure 3. Average HHI and MHHI for Product and Labor Market. Source: Authors' calculation using Compustat and Thomson 13F Institutional ownership data.

Solving the Model and Obtaining Labor and Capital Shares

Once we have calibrated the model parameters, we can solve the model for K^* and calculate the labor and capital shares. We solve the model for each year using the corresponding values of the concentration and common ownership parameters, which vary over time, and keeping constant the rest of the model parameters.

For each year, we solve the model numerically using a grid for K of 100,000 values between 0 and E (one can also think of this as equivalent to a grid for the savings rate $s = K/E$ of 100,000 values between 0 and 1). For each value of K in the grid, we calculate the value of the labor and capital markdowns:

$$1 + \mu_L = \frac{1 + H_{labor}/\eta}{1 - (H_{product} - \lambda_{inter}) (1 - 1/N) / \theta'}$$

$$1 + \mu_K = \frac{1 + H_{capital} / (\varepsilon(K) (1 - (1 - \delta) / \rho(K)))}{1 - (H_{product} - \lambda_{inter}) (1 - 1/N) / \theta'}$$

where $\rho(K) = \left(\frac{K}{E-K}\right)^{\frac{\gamma}{1-\gamma}} \beta^{-\frac{1}{1-\gamma}}$ is the real interest rate, and $\varepsilon(K) = \frac{\rho(K)}{\rho'(K)K} = \frac{1-\gamma}{\gamma} (1 - K/E)$, is its elasticity with respect to K .

We then calculate the level of labor corresponding to each value of K in the grid, using the following equation which is obtained by combining the labor supply of the workers with the first-order condition of the firms with respect to labor:

$$L = \left[\frac{A\alpha}{\chi^{\frac{1}{1-\sigma}} (1 + \mu_L)} \right]^{\frac{1}{1-\alpha+\frac{1}{\eta}}} K^{\frac{1-\alpha}{1-\alpha+\frac{1}{\eta}}}$$

We can now replace (again, for each element of the grid) this value for L in the first-order condition for capital:

$$\frac{A(1-\alpha)(K/L)^{-\alpha}}{(1 + \mu_K)(r - 1 + \delta)} - 1 = 0.$$

We solve for the equilibrium K^* by finding the value of K , among the 100,000 possible values in the grid, that yields the minimum value of the left-hand side of this equation squared.

Finally, we evaluate the formulas for the equilibrium labor and capital shares in the model at K^* :

$$\text{Labor Share}^* = \frac{\alpha}{1 + \mu_L^*} = \alpha \frac{1 - (H_{product} - \lambda_{inter}) (1 - 1/N) / \theta}{1 + H_{labor} / \eta}.$$

$$\text{Capital Share}^* = \frac{1 - \alpha}{1 + \mu_K^*} = (1 - \alpha) \frac{1 - (H_{product} - \lambda_{inter}) (1 - 1/N) / \theta}{1 + H_{capital} / (\varepsilon(K^*) (1 - (1 - \delta) / \rho(K^*)))}.$$

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